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MCQ-003-001543

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

May / June - 2018

Statistics

(S-502 : Mathematical Statistics) (New Course)

Faculty Code : 003

Subject Code : 001543

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Q. No. **1** carries **20** marks, Q. No. **2** and **3** each carries **25** marks.

(2) Students can use their own scientific calculator.

1 Filling the blanks and short questions : (Each **1** mark) **20**

(1) ____________ is a moment generating function of Chi-square distribution.

(2) For Normal distribution $\mu_4 = k_4 + 3k_2^2$ is ____________.

(3) If two independent variates $X_1 \sim \Lambda(\mu_1, \sigma_1^2)$ and $X_2 \sim \Lambda(\mu_2, \sigma_2^2)$ then $X_1 \cdot X_2$ is distributed as __________.

(4) Weibull distribution has application in ____________.

(5) The range of multiple correlation coefficient R is ____________.

(6) Define Log Normal with $\log_e(x-a)$ distribution.

(7) Write mean and variance of Weibul distribution.

(8) Define Caushi's distribution.

(9) ____________ is a characteristic function of Poisson distribution.

(10) ____________ is a moment generating function of Normal distribution.

(11) For Normal distribution μ_{2n} = ____________.

(12) If two independent variates $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ then $X_1 - X_2$ is distributed as ____________.

(13) If two independent variates $X_1 \sim y(n_1)$ and $X_2 \sim y(n_2)$ then $\frac{x_1}{x_1 + x_2}$ is distributed as ____________.

(14) The range of partial regression coefficient is ____________.

(15) Write mean and variance of Gama distribution with parameter (α, p).

(16) Write mean and variance of Lapace (double) exponential distribution.

(17) If χ_1^2 and χ_2^2 are two independent Chi-square variates with d.f. n_1 and n_2, respectively, then the distribution of $\frac{\chi_1^2}{\chi_2^2}$ is ____________.

(18) ____________ is a moment generating function of $\gamma(\alpha, p)$.

(19) ____________ is a characteristic function of Standard Normal distribution.

(20) ____________ is a characteristic function of Geometric distribution.

2 (a) Write the answer any THREE : (Each **2** marks) **6**

(1) Define Weibul distribution.

(2) Prove that $b_{12.3} = \frac{b_{12} - b_{13}b_{23}}{1 - b_{13}b_{23}}$.

(3) If $u = \frac{x-a}{h}$, a and h being constants then $\varnothing_u(t) = e^{(-iat/h)}\varnothing_x(t/h)$.

(4) In trivariate distribution it is found that $r_{12} = 0.77, r_{13} = 0.72$ and $r_{23} = 0.52$ Find (i) $r_{12.3}$ (ii) $R_{1.23}$.

(5) Prove that $\varnothing_u(t) = e^{-i\mu t}\varnothing_x(\text{t})$; where $u = x - \mu$.

(6) Why characteristic function need ?

(b) Write the answer any **THREE** : (Each **3** marks) **9**

(1) Prove that $\mu_r' = (-i)^r \left[\frac{d^r}{dt^r} \varnothing_x(t) \right]_{t=0}$.

(2) Define Exponential distribution and obtain its MGF. From MGF obtain its mean and variance.

(3) Obtain probability density function for the characteristic function $\varnothing_x(t) = e^{-(\frac{1}{2}t^2\sigma^2)}$.

(4) Obtain probability density function for the characteristic function $\varnothing_x(t) = p(1-qe^{it})^{-1}$.

(5) Usual notation prove that $\sigma_{1.23}^2 = \sigma_1^2(1-r_{12}^2)(1-r_{13.2}^2)$.

(6) Obtain mean and variance of Beta distribution of first kind.

(c) Write the answer any **TWO** : (Each **5** marks) **10**

(1) Derive χ^2 distribution and show that $2\beta_2 - \beta_1 - 6 = 0$.

(2) Derive t-distribution.

(3) Usual notation of multiple correlation and multiple regression, prove that $R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{13}}{1 - r_{23}^2}$.

(4) Obtain marginal distribution of x for Bi-variate distribution.

(5) State and Prove that Chebchev's inequality.

3 (a) Write the answer any **THREE** : (Each **2** marks) **6**

(1) Define Beta-I and Beta-II distribution.

(2) Prove that $\varnothing_x(0) = 1$ and $|\varnothing_x(t)| \le 1$.

(3) Usual notion of multiple correlation and multiple regression, prove that $\sum X_{1.2} X_{3.12} = 0$.

(4) Define truncated distribution.

(5) Obtain characteristic function of Poisson distribution with parameter λ.

(6) In trivariate distribution it is found that

$\sigma_1 = 2, \sigma_2 = \sigma_3 = 3, r_{12} = 0.7, r_{23} = r_{31} = 0.5$

Find (i) $b_{13.2}$ (ii) $\sigma_{3.12}$.

(b) Write the answer any **THREE** : (Each **3** marks) **9**

(1) Prove that $\mu_r = (-i)^r \left[\frac{d^r}{dt^r} \varnothing_x(t) \right]_{t=0}$

(2) Obtain MGF of Normal distribution.

(3) Obtain mean and variance of Beta distribution of second kind.

(4) Obtain mean and variance of Uniform Distribution.

(5) Usual notation of multiple correlation and multiple regression, prove that $b_{12.3}b_{23.1}b_{31.2} = r_{12.3}r_{23.1}r_{31.2}$.

(6) Usual notation prove that $r_{12.3}\frac{\sigma_{1.3}}{\sigma_{2.3}} = b_{12.3}$

(c) Write the answer any **TWO** : (Each **5** marks) **10**

(1) Derive Normal distribution.

(2) Derive F-distribution.

(3) Usual notation of multiple correlation and multiple regression, prove that $r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1-r_{13}^2)(1-r_{23}^2)}}$.

(4) Obtain relation between F and χ^2.

(5) If the joint pdf of x and y is

$$f(x,y) = \frac{1}{2\pi} e^{-\frac{1}{2(1-p^2)}\left\{x^2 = -2pxy + y^2\right\}}$$

where $-\infty \le x, y \le \infty; -1 \le p \le 1$

then find (i) Marginal distribution of y. (ii) Conditional distribution of x when y is given.